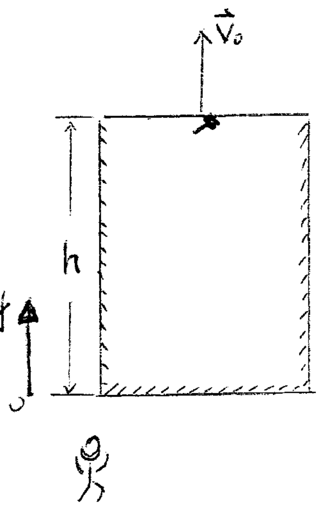
Physics 4A Exam I
Fall 2009

Name: Solutions
Lab (Circle one): 61 62 63

1. (25 points) An elevator is moving upward at a constant speed of V_o. A bolt in the elevator ceiling h meter above the elevator floor works loose and falls. (a) How long does it take for the bolt to the elevator floor. According to the observer standing on the lobby of the building (b) what is the velocity of the bolt just as it hits the elevator floor and (c) what distance did the bolt travel between the ceiling and the elevator floor.



a). at time t, the bolt hits the floor.

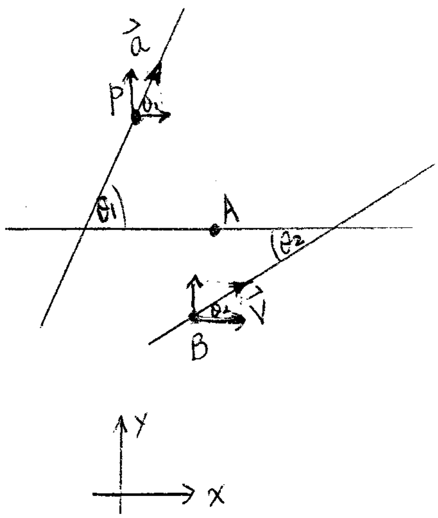
$$y_b = y_o + v_o t + \frac{1}{2} a t^2$$
$$= h + v_o t - \frac{1}{2} g t^2$$
$$y_e = y_o + v_o t + \frac{1}{2} a t^2 = v_o t$$
$$y_b = y_e$$
$$h + v_o t - \frac{1}{2} g t^2 = v_o t$$
$$\boxed{t = \sqrt{\frac{2h}{g}}}$$

Velocity: b) $V_b = V_o + at$
$$= V_o - gt$$
$$= V_o - g\sqrt{\frac{2h}{g}}$$
$$\boxed{V_b = V_o - \sqrt{2gh}} \text{ downward.}$$

Position: c). $y_b = h + V_o\sqrt{\frac{2h}{g}} - \frac{1}{2} g \sqrt{\frac{2h}{g}}^2$
$$= h + V_o\sqrt{\frac{2h}{g}} - h$$
$$= V_o\sqrt{\frac{2h}{g}}$$

distance traveled $\left| h - V_o\sqrt{\frac{2h}{g}} \right|$

2. (25 points) Jeeps P and B race along straight lines, across flat terrain, and past stationary border guard A. Relative to the guard, B travels at a constant speed of V = 5 m/s at the angle θ_2, P accelerates from rest at a constant rate of a = 2 m/s^2 at the angle θ_1. At time t, what are the (a) velocity of P relative to B, and (b) acceleration of P relative to B?



P: $\vec{V}_P = V_x \hat{i} + V_y \hat{j}$

$V_x = V_{ox} + a_x \cdot t = a\cos\theta_1 t$

$V_y = V_{oy} + a_y t = a\sin\theta_1 t$

$\vec{V}_P = 2\cos\theta_1 t\, \hat{i} + 2\sin\theta_1 t\, \hat{j}$

$\vec{a}_P = \frac{d\vec{v}_P}{dt} = 2\cos\theta_1 \hat{i} + 2\sin\theta_1$

B: $\vec{V}_B = V_x \hat{i} + V_y \hat{j}$

$V_x = V_{ox} + a_x \cdot t = V\cos\theta_2$

$V_y = V_{oy} + a_y \cdot t = V\sin\theta_2$

$\vec{V}_B = 5\cos\theta_2 \hat{i} + 5\sin\theta_2 \hat{j}$

$\vec{a}_B = \frac{d\vec{V}_B}{dt} = 0$

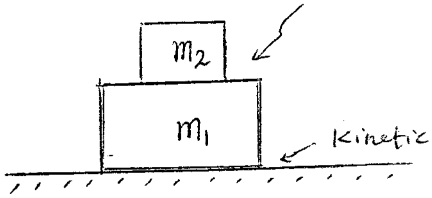
$\vec{V}_{Pg} = \vec{V}_{PB} + \vec{V}_{Bg}$

$\vec{V}_{PB} = \vec{V}_{Pg} - \vec{V}_{Bg} = \vec{V}_P - \vec{V}_B = (2\cos\theta_1 t - 5\cos\theta_2)\hat{i} + (2\sin\theta_1 t - 5s$

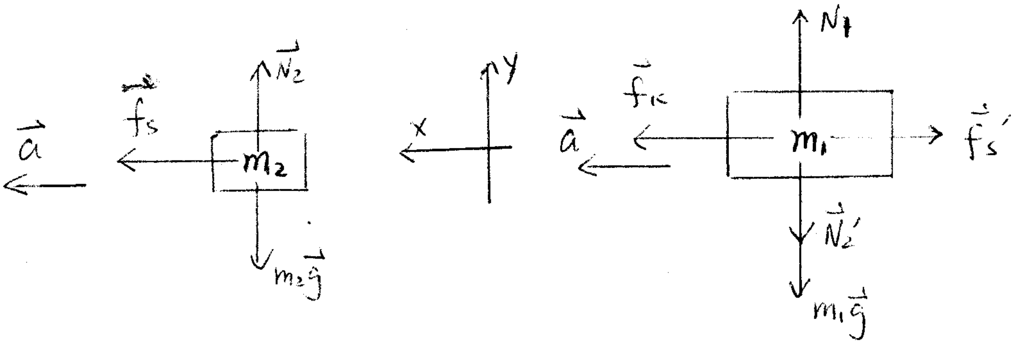
$\vec{V}_{PB} = (2\cos\theta_1 t - 5\cos\theta_2)\hat{i} + (2\sin\theta_1 t - 5\sin\theta_2)\hat{j}$

$\vec{a}_{PB} = \vec{a}_P - \vec{a}_B = 2\cos\theta_1 \hat{i} + 2\sin\theta_1 \hat{j}$

3. (25 points) Mass m_2 is on the top of mass m_1, and coefficient of static friction between the two masses is μs. They are sliding on a rough horizontal tabletop. Find the maximum coefficient of kinetic friction μk such that m_2 does not slip on m_1.



m_1, m_2 move together with acceleration a slowing down, finally stop.



$N_2' = N_2$

$f_s' = f_s$

m_2 has a tendency to move rightward so static friction by m_1 on m_2 leftward.

$$\begin{cases} f_s = m_2 a \\ N_2 - m_2 g = 0. \end{cases}$$

$$N_2 = m_2 g$$

$$f_{smax} = \mu_s N_2 = \mu_s m_2 g = m_2 a$$

$$\therefore \boxed{a = \mu_s g}$$

$$\begin{cases} f_K - f_s' = m_1 a \\ N_1 - N_2' - m_1 g = 0 \end{cases}$$

$$N_1 = N_2 + m_1 g = (m_1 + m_2) g$$

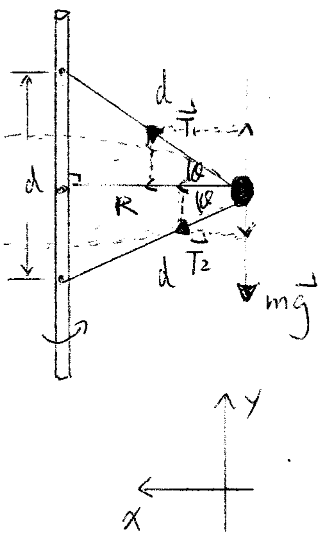
$$f_K = f_s' + m_1 a$$

$$\mu_K N_1 = m_2 a + m_1 a$$

$$\mu_K (m_1 + m_2) g = (m_1 + m_2) \mu_s$$

$$\boxed{\mu_K = \mu_s}$$

4. (25 points) A ball of mass m is attached to a vertical rod by two strings. The strings are separated a distance d on the rod and both strings are length d. When the system rotates about the axis of the rod, the strings are extended and the tension in the upper string is twice the tension in the lower string. Find (a) the velocity of the ball and (b) the minimum velocity of the ball. (extra credit)



The ball moves in a circular motion of radius R. $R = d\cos\theta$

x: $T_1\cos\theta + T_2\cos\theta = m\frac{v^2}{R}$ —— (1)

y: $T_1\sin\theta - T_2\sin\theta - mg = 0$ —— (2)

$T_1 = 2T_2$. $\theta = 30°$.

from (2) $2T_2\sin 30° - T_2\sin 30° = mg$

$T_2 \sin 30° = mg$

$T_2 = 2mg$ into (1)

$3T_2\cos 30° = m\frac{v^2}{R}$

$3\cdot 2mg\cdot\cos 30°\cdot R = mv^2$

$v^2 = 6\cdot\frac{\sqrt{3}}{2}gd\cdot\cos 30° = 6\times\frac{\sqrt{3}}{2}gd\frac{\sqrt{3}}{2}$

$$\boxed{v = \sqrt{\frac{9}{2}gd}}$$

minimum velocity when $T_2 = 0$.

x: $T_1\cos 30° = m\frac{v_{min}^2}{R}$

y: $T_1\sin 30° - mg = 0$ $T_1\sin 30° = mg$ $T_1 = 2mg$.

$2mg\cdot\cos 30° = m\frac{v_{min}^2}{R}$

$v_{min}^2 = 2gR\cos 30° = 2gd\cos^2 30° = 2gd\frac{3}{4}$ $\boxed{v_{min} = \sqrt{\frac{3}{2}gd}}$