

Give complete solutions to the following problems. Be sure to provide all the necessary steps to support your answers.

1. let W be a the set of all points on a line parallel to $\mathbf{v} = 4\mathbf{i} + 3\mathbf{j}$ and passing through the origin.
 - a. Show that W is vector subspace of \mathcal{R}^2 .

- b. Find the orthogonal complement of W .

2. Let $S = \{(-1, 4, 3), (1, 1, 0)\}$, $\mathbf{u} = (-1, 1, 0)$
Find the projection of \mathbf{u} onto the linear span of S

3. Let $S = \{(-4, -1, 1), (0, 1, 1)\}$, $\mathbf{x} = (-1, 1, 0)$
Express \mathbf{x} as a linear combination of two vectors, one in the linear span of S and another in the orthogonal complement of span S .

4. Let $S = \{(1, 1, 0, -1), (0, -1, 1, -1), (1, 0, 1, 1)\}$, $\mathbf{x} = (3, 4, 5, 6)$

Express \mathbf{x} as a linear combination of two vectors, one in the linear span of S and another in the orthogonal complement of span S .

5. Let $S = \{(-1, 1, 1), (1, 0, 1)\}$, $\mathbf{x} = (1, 2, 3)$

Find the projection of \mathbf{x} onto the linear span of S

a. using vector projection.

b. using a projection matrix U whose columns are the orthogonal basis vectors in S .

6. Find the distance from \mathbf{v} to the plane in \mathbb{R}^3 spanned by the vectors \mathbf{u}_1 and \mathbf{u}_2 .
 $\mathbf{v} = (3, 5, 2)$, $\mathbf{u}_1 = (1, -1, 2)$, $\mathbf{u}_2 = (1, 2, 1)$